# Evaluation of Loss Factor of Multilayered Inhomogeneous Waveguides for Magnetostatic Waves Using Efficient Finite Element Formalism

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***Abstract* —An efficient finite element solution procedure is developed for calculating propagation losses of magnetostatic waves in multilayered inhomogeneous waveguides. The final matrix equation is reduced to a standard complex eigenvalue problem whose eigenvalue corresponds to the complex phase constant itself. Thus, iteration procedures are not necessary and the phase and attenuation constants can be directly obtained by solving a standard eigenvalue equation. The validity of the method is confirmed by calculating propagation losses of magnetostatic surface waves in a single YIG-film structure. Numerical results for a triple-layered YIG-film structure are also presented. It is found that in the triple-layered structure, propagation losses are highly dependent on the line width of the film in which the magnetostatic potential is well confined.**

## I. Introduction

MAGNETOSTATIC wave (MSW) propagation losses for a homogeneous single YIG-film structure have been extensively investigated [1]–[7]. It is known that at high frequencies, a magnetostatic surface wave (MSSW) propagation loss factor $L$ (dB/$\mu$s) for a single YIG-film structure is proportional to the frequency with a slope of $4\pi(76.4\lambda/\gamma^2\mu_0^2 M_s)$ and can be written as $L \simeq 76.4\Delta H$ [2]–[5]. Here $\lambda$ is the damping parameter, $M_s$ is the magnetization, $\gamma$ is the gyromagnetic ratio, and $\Delta H$ is the ferromagnetic resonance line width. It has also been found that magnetostatic forward volume wave (MSFVW) propagation loss characteristics for a single YIG-film structure are similar to MSSW modes and that the dependence of loss factor on line width is also $76.4\Delta H$ [6]. Stancil [7] has investigated propagation losses of MSSW, MSFVW, and magnetostatic backward volume waves (MSBVW) in detail.

Recently, much attention has been paid to MSW modes in various complicated structures such as multilayered and inhomogeneous films to improve the delay characteristics [6], [8]–[10], and numerical approaches such as the variational method [9] and the finite element method [10] have been introduced for analyzing nonuniform geometries. These methods are valid for the solution of multilayered inhomogeneous waveguiding structures. However, the final matrix equation should be solved by searching for the phase constant such that the determinant of the matrix vanishes. Since the phase constant for lossy waveguides becomes complex, it is very difficult to solve the above matrix equation so as to have the determinant of the complex matrix vanish by searching for two values, i.e., the phase constant and the attenuation constant.

In this paper, an efficient finite element solution procedure is developed for calculating MSW propagation losses for multilayered inhomogeneous waveguides. The final matrix equation is reduced to a standard complex eigenvalue problem whose eigenvalue corresponds to the complex phase constant itself. Thus, iteration procedures are not necessary and the phase and attenuation constants can be directly obtained by solving a standard eigenvalue equation.

The validity of the method is confirmed by calculating MSSW propagation losses for a single YIG-film structure. Also, MSBVW propagation losses for a single YIG-film structure are calculated. It is shown that the MSBVW propagation loss characteristic is similar to that for MSFVW modes. Furthermore, MSFVW propagation losses for a triple-layered YIG-film structure are investigated. It is found that in the triple-layered structure, propagation losses depend greatly on the line width of the film in which the magnetostatic potential is well confined.

## II. Basic Equations

We consider the multilayered inhomogeneous waveguide for MSW modes shown in Fig. 1. When the bias field $H_0$ is applied in parallel with the $x$, $y$, and $z$ directions, MSFVW, MSBVW, and MSSW modes propagate along the $y$ direction, respectively.

With a time dependence of the form $\exp(j\omega t)$ being implied, the relative permeability tensor $[\mu_r]$ takes the

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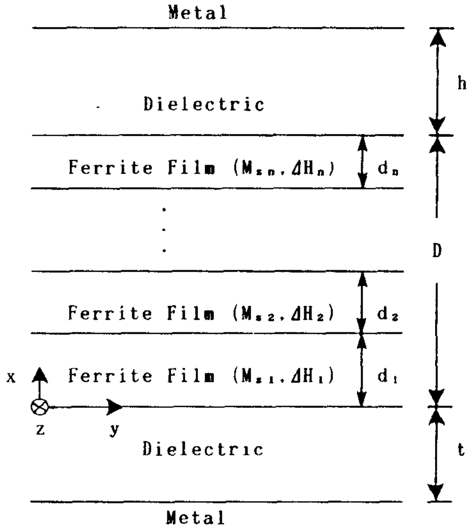


Fig. 1. Geometry of a planar MSW waveguide.

form

$$[\mu_r] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mu & j\kappa \\ 0 & -j\kappa & \mu \end{bmatrix} \quad \text{for MSFVW} \qquad (1a)$$

$$[\mu_r] = \begin{bmatrix} \mu & 0 & -j\kappa \\ 0 & 1 & 0 \\ j\kappa & 0 & \mu \end{bmatrix} \quad \text{for MSBVW} \qquad (1b)$$

$$[\mu_r] = \begin{bmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{for MSSW} \qquad (1c)$$

where

$$\mu = 1 + \frac{\omega_i \omega_m}{\omega_i^2 - \omega^2} \qquad (2)$$

$$\kappa = \frac{\omega_m \omega}{\omega_i^2 - \omega^2} \qquad (3)$$

$$\omega_i = -\gamma\mu_0 H_i \left(1 + j\frac{\lambda\omega}{\gamma^2\mu_0^2 H_i M_s}\right) \qquad (4)$$

$$\omega_m = -\gamma\mu_0 M_s. \qquad (5)$$

Here $H_i$ is the internal magnetic field in the ferrimagnetic film; $H_i = H_0 - M_s$ for MSFVW and $H_i = H_0$ for MSBVW and MSSW. At a given frequency, the damping parameter $\lambda$ is related to the line width $\Delta H$ through the expression [3]

$$\lambda = \frac{\gamma^2\mu_0^2 M_s}{2\omega}\Delta H \qquad (6)$$

and $\Delta H$ can be measured from a ferrimagnetic resonance (FMR) absorption.

Assuming that there is no variation of all fields in the $z$ direction, from Maxwell's equations and the magnetostatic approximation condition the following basic equations for MSW waveguides are obtained:

$$\partial B_x/\partial x - jskB_y = 0 \qquad (7)$$

$$H_x = -\partial\phi/\partial x \qquad (8)$$

$$H_y = jsk\phi \qquad (9)$$

where $k$ is the complex phase constant in the $y$ direction, $\mathrm{Re}(k) \equiv \beta$ and $\mathrm{Im}(k) \equiv -\alpha$ are the phase and attenuation constants, respectively, $s = \pm 1$ is a directional parameter [8], [9], and $\phi$ is the magnetostatic potential. The time delay $T$ is given by

$$T = \frac{d\beta}{d\omega} \quad (\text{s/m}). \qquad (10)$$

The loss factor $L$ in dB/$\mu$s is defined as

$$L = [8.686\alpha/T] \times 10^{-6} \quad (\text{dB}/\mu\text{s}). \qquad (11)$$

## III. Mathematical Formulations

We consider the following four cases in Fig. 1:

1) $t$ and $h$ are finite;
2) $h$ is finite and $t \to \infty$;
3) $t$ is finite and $h \to \infty$;
4) $t \to \infty$ and $h \to \infty$.

Dividing the region $-t \leqq x \leqq D + h$, $0 \leqq x \leqq D + h$, $-t \leqq x \leqq D$, or $0 \leqq x \leqq D$ into a number of second-order line elements [11] for case 1), 2), 3), or 4), respectively, using the finite element method based on a Galerkin procedure on (7), integrating by parts, and assembling the complete matrix for the region by adding the contributions of all different elements [10], we obtain

$$k^2[A]\{\phi\} + k[B]\{\phi\} + [C]\{\phi\} + k\{\tilde{\phi}\} = \{0\} \qquad (12)$$

where

$$[A] = \begin{cases} \sum_e \int_{x_1}^{x_2} \mu_e \{N\}\{N\}^T dx & \text{for MSSW and MSFVW} \\ \sum_e \int_{x_1}^{x_2} \{N\}\{N\}^T dx & \text{for MSBVW} \end{cases} \qquad (13)$$

$$[B] = \begin{cases} [0] & \text{for MSFVW and MSBVW} \\ s\sum_e \int_{x_1}^{x_2} \kappa_e\left(\{N_x\}\{N\}^T + \{N\}\{N_x\}^T\right) dx & \text{for MSSW} \end{cases} \qquad (14)$$

$$[C] = \begin{cases} \sum_e \int_{x_1}^{x_2} \mu_e \{N_x\}\{N_x\}^T dx & \text{for MSBVW and MSSW} \\ \sum_e \int_{x_1}^{x_2} \{N_x\}\{N_x\}^T dx & \text{for MSFVW} \end{cases} \qquad (15)$$

$$\{\tilde{\phi}\} = \begin{cases} [0 \quad 0 \quad \cdots \quad 0 \quad 0]^T & \text{for 1)} \\ [\phi_0 \quad 0 \quad \cdots \quad 0 \quad 0]^T & \text{for 2)} \\ [0 \quad 0 \quad \cdots \quad 0 \quad \phi_D]^T & \text{for 3)} \\ [\phi_0 \quad 0 \quad \cdots \quad 0 \quad \phi_D]^T & \text{for 4).} \end{cases} \qquad (16)$$

TABLE I
EXACT AND FINITE ELEMENT SOLUTIONS FOR MSSW MODES IN A SINGLE YIG-FILM STRUCTURE

| f (GHz) | ΔH (Oe) | s | $k = \beta - j\alpha$ cm$^{-1}$ | | | |
|---|---|---|---|---|---|---|
| | | | $\beta_{exact}$ | $\alpha_{exact} \times 10^2$ | $\beta_{FEM}$ | $\alpha_{FEM} \times 10^2$ |
| 2.1 | 0.1 | 1 | 12.1316 | 0 699061 | 12 1316 | 0.699053 |
| | | -1 | 26.5995 | 1 51969 | 26 5995 | 1.51969 |
| | 0.3 | 1 | 12 1316 | 2 09716 | 12.1316 | 2 09716 |
| | | -1 | 26 5995 | 4 55908 | 26.5995 | 4 55908 |
| 2 5 | 0 1 | 1 | 206.266 | 4.28496 | 206 267 | 4 28506 |
| | | -1 | 231.271 | 3.25830 | 231 271 | 3 25831 |
| | 0.3 | 1 | 206 266 | 12.8549 | 206 267 | 12.8552 |
| | | -1 | 231.271 | 9 77491 | 231.271 | 9.77494 |
| 3 0 | 0.1 | 1 | 890.659 | 14.1368 | 893.671 | 14.4214 |
| | | -1 | 890.669 | 14.1351 | 890 763 | 14.1417 |
| | 0.3 | 1 | 890.659 | 42 4102 | 893 671 | 43.2642 |
| | | -1 | 890.699 | 42.4052 | 890.763 | 42 4251 |

Here $T$, $\{\cdot\}$, and $\{\cdot\}^T$ denote a transpose, a column vector, and a row vector, respectively. The components of the $\{\phi\}$ vector are the values of $\phi$ at the nodal points in the region divided into elements; $\phi_0$ and $\phi_D$ are the values of $\phi$ at the nodal points on $x=0$ and $x=D$, respectively; $\{N\}$ is the shape function vector, $\{N_x\}=d\{N\}/dx$; $\{0\}$ is a null vector; $[0]$ is a null matrix; $x_1$ and $x_2$ are the $x$ coordinates of the two ends of the line element ($x_1<x_2$); and $\Sigma_e$ extends over all different elements.

In order to obtain directly the complex phase constant $k$, (12) is reduced to a standard complex eigenvalue equation [11]:

$$\begin{bmatrix} [0] & [U] \\ -[A]^{-1}[C] & -[A]^{-1}[\tilde{B}] \end{bmatrix} \begin{bmatrix} \{\phi\} \\ k\{\phi\} \end{bmatrix} = k \begin{bmatrix} \{\phi\} \\ k\{\phi\} \end{bmatrix} \tag{17}$$

where $[U]$ is a unit matrix and $[\tilde{B}]$ is given by

$$[\tilde{B}] = [B] + [D]. \tag{18}$$

Here

$$[D] = \begin{bmatrix} D_{11} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & D_{nn} \end{bmatrix} \tag{19}$$

$$D_{11}=0 \qquad D_{nn}=0 \qquad \text{for 1)} \tag{20a}$$

$$D_{11}=1 \qquad D_{nn}=0 \qquad \text{for 2)} \tag{20b}$$

$$D_{11}=0 \qquad D_{nn}=1 \qquad \text{for 3)} \tag{20c}$$

$$D_{11}=1 \qquad D_{nn}=1 \qquad \text{for 4)} \tag{20d}$$

where $n$ corresponds to the number of nodal points. The final matrix equation (17) is a standard eigenvalue problem whose eigenvalue corresponds to the complex phase constant $k$ itself. Thus, iteration procedures are not necessary and the phase and attenuation constants can be directly obtained by solving the standard eigenvalue equation (17).

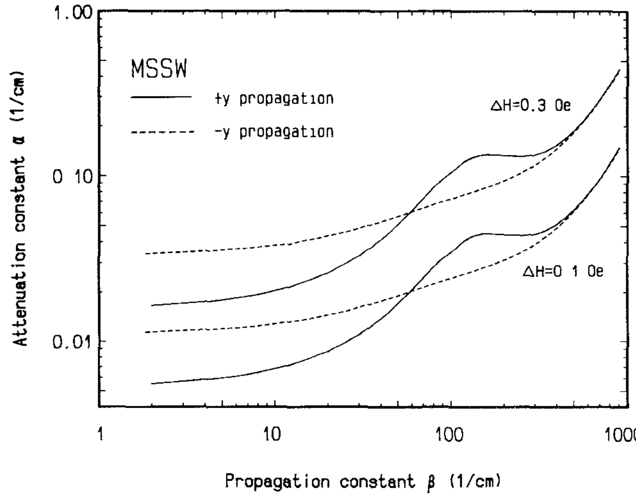


Fig. 2. Attenuation constant as a function of propagation constant for MSSW modes in a single YIG-film structure.

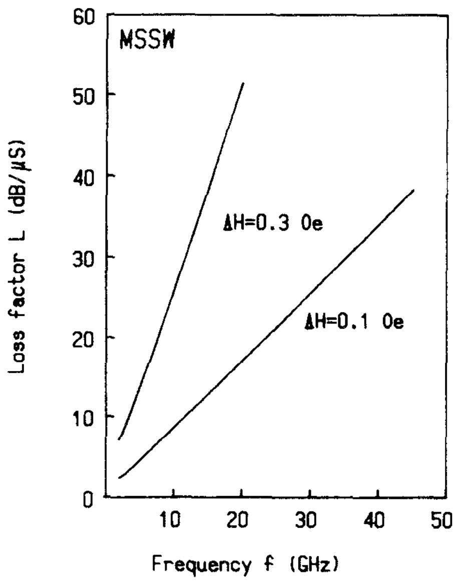


Fig. 3 Frequency dependence of loss factor for MSSW modes in a single YIG-film structure.

## IV. COMPUTED RESULTS

As a test for the numerical accuracy of the method proposed here, we first consider a single YIG-film structure [3]. Table I shows the results for MSSW modes where $H_0 = 255$ Oe, $M_s = 1750$ G, $d = 10$ μm, $h = 75$ μm, and $t \to \infty$; $\beta_{\text{exact}}$ and $\alpha_{\text{exact}}$ are the exact solutions [3]; and $\beta_{\text{FEM}}$ and $\alpha_{\text{FEM}}$ are the finite element solutions. The number of elements is 9. The damping parameter $\lambda$ is calculated from an FMR line width of 0.1 and 0.3 Oe at $f = 9.3$ GHz and it is assumed that $\lambda$ is independent of frequency [3], [5], [6]. Our results agree well with the exact solutions [3] for both phase and attenuation. Fig. 2 shows a plot of $\alpha$ as a function of $\beta$. The frequency dependence of the loss factor $L$ is given in Fig. 3. The results in Figs. 2 and 3 are also in agreement with those in [3, figs. 3 and 5], respectively.

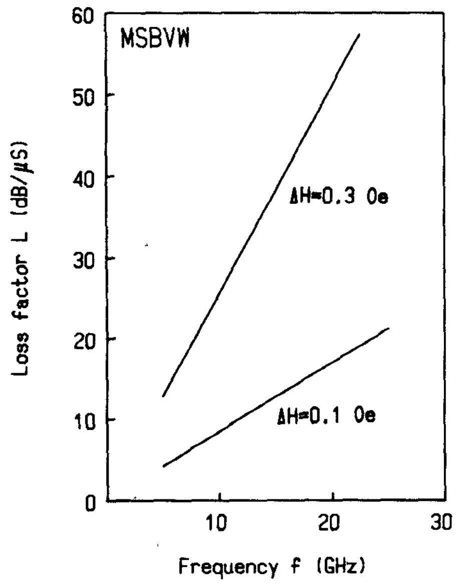


Fig. 4. Frequency dependence of loss factor for MSBVW modes in a single YIG-film structure.

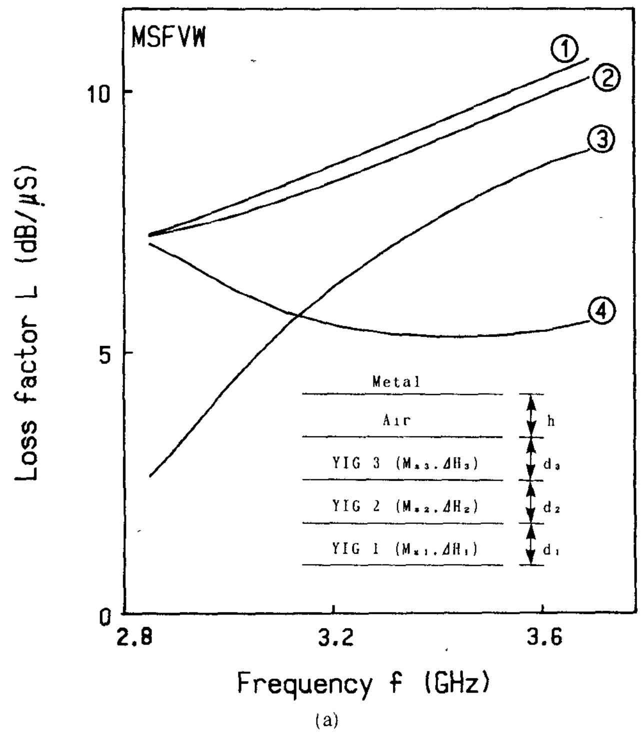


Fig. 5. MSFVW modes in a triple-layered YIG-film structure. (a) Frequency dependence of loss factor. (*Continued in next column*)

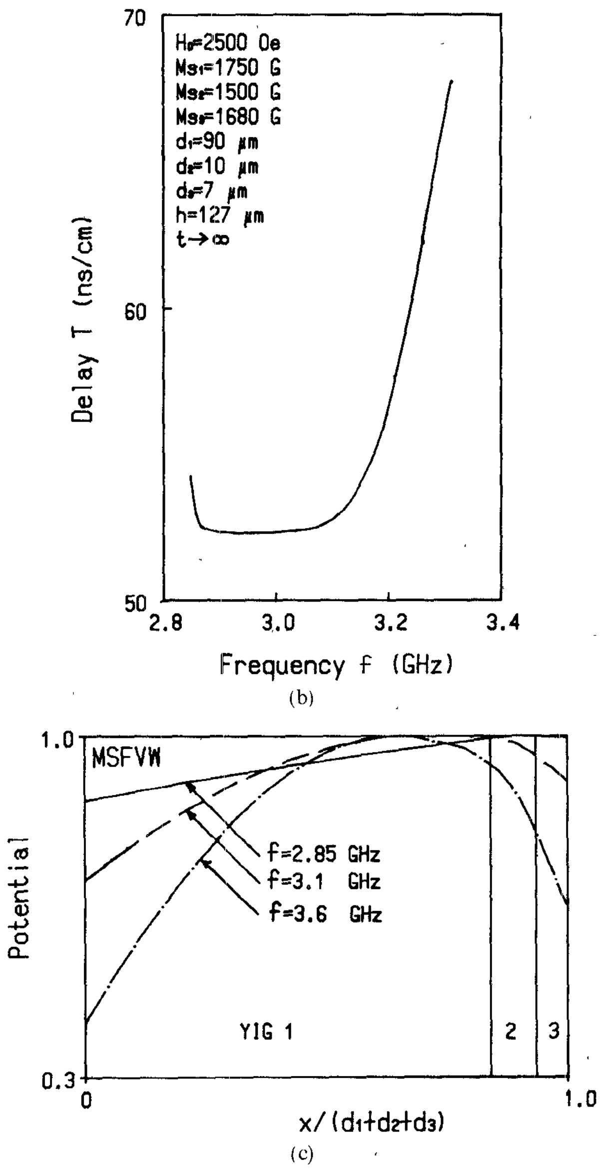


Fig. 5. (*Continued*) (b) Group delay. (c) Potential profile.

Next, we consider MSBVW propagation losses for a single YIG-film structure. Fig. 4 shows a plot of the loss factor $L$ as a function of frequency from 5 to 30 GHz, where $M_s = 1200$ G, $d = 30$ $\mu$m, $h = 635$ $\mu$m, $t \to \infty$, and $\Delta H = 0.1$ and 0.3 Oe at $f = 9$ GHz. It is shown by Fig. 4 that propagation losses increase rapidly with frequency and that the dependence of the loss factor $L$ on $\Delta H$ is also $76.4\Delta H$. This is similar to that for MSFVW modes [6].

Lastly, we consider MSFVW propagation losses for a triple-layered YIG-film structure which have not been fully investigated [6], [8]. The frequency dependence of $L$, the group delay $T$, and potential profiles are shown in Fig. 5(a), (b), and (c), respectively, where $H_0 = 2500$ Oe, $M_{s1} = 1750$ G, $M_{s2} = 1500$ G, $M_{s3} = 1680$ G, $d_1 = 90$ $\mu$m, $d_2 = 10$ $\mu$m, $d_3 = 7$ $\mu$m, $h = 127$ $\mu$m, $t \to \infty$, and $\Delta H_1$, $\Delta H_2$, $\Delta H_3$, are given by

$$\Delta H_1 = \Delta H_2 = \Delta H_3 = 0.3 \text{ Oe} \qquad \text{for ①} \quad (21a)$$

$$\Delta H_3 = 0.1 \text{ Oe} \qquad \Delta H_1 = \Delta H_2 = 0.3 \text{ Oe} \qquad \text{for ②} \quad (21b)$$

$$\Delta H_2 = 0.1 \text{ Oe} \qquad \Delta H_3 = \Delta H_1 = 0.3 \text{ Oe} \qquad \text{for ③} \quad (21c)$$

$$\Delta H_1 = 0.1 \text{ Oe} \qquad \Delta H_2 = \Delta H_3 = 0.3 \text{ Oe} \qquad \text{for ④}. \quad (21d)$$

For comparison with the lossless case $\Delta H_1 = \Delta H_2 = \Delta H_3 = 0$ [8], the results above 2.8 GHz are presented in Fig. 5. The delay curve and potential profiles are almost the same for all the cases 1, 2, 3, and 4. Also, the delay curve in Fig. 5(b) is like that in [8, fig. 3]. When three films have the same line width (case 1), the frequency depen-

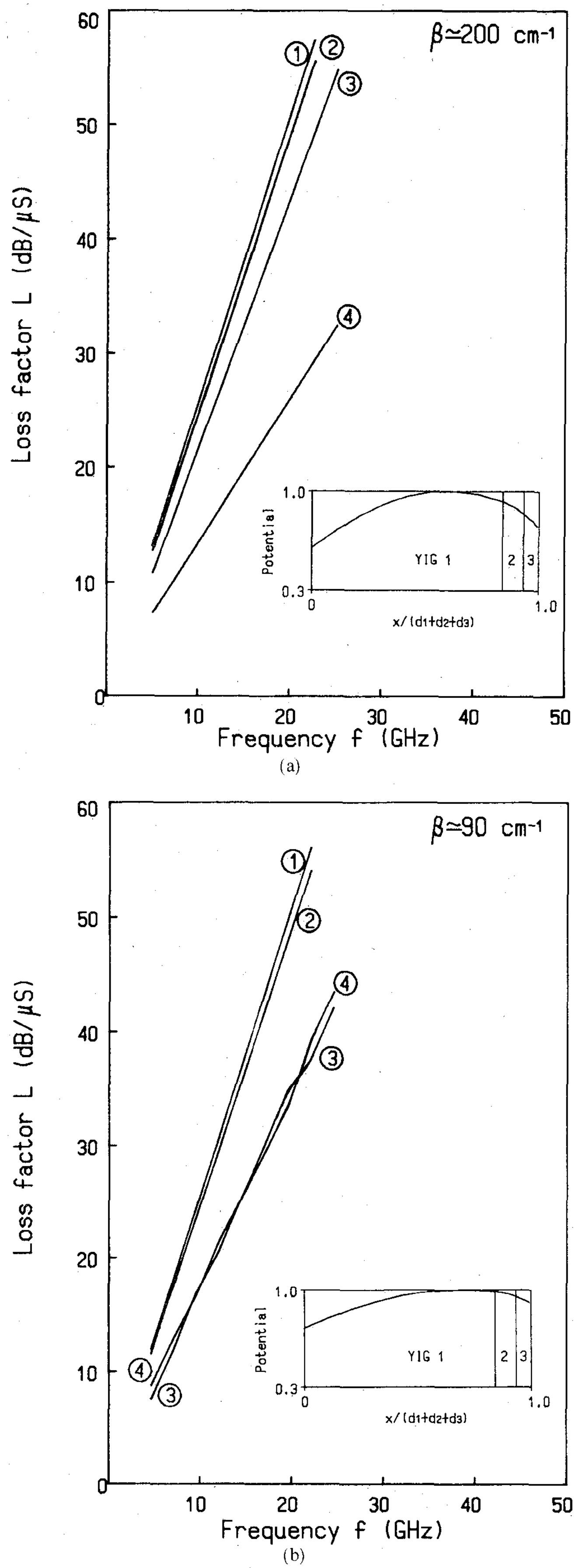


Fig. 6. Frequency dependence of loss factor for MSFVW modes in a triple-layered YIG-film structure. (a) $\beta \doteqdot 200\ \mathrm{cm}^{-1}$. (b) $\beta \doteqdot 90\ \mathrm{cm}^{-1}$.

dence of $L$ is similar to that for a single YIG-film structure. With increasing frequency, the potential is well confined in the first film. Thus, the loss factor $L$ for case 4 with smaller $\Delta H_1$ decreases with frequency. In cases 2 and 3 with larger $\Delta H_1$, on the other hand, the loss factor $L$ increases with frequency. Since the magnitude of the potential is relatively small in the third film, the influence of the value of $\Delta H_3$ on $L$ is very small and the difference between the loss factors of cases 1 and 2 is small. It is found from Fig. 5(a) and (b) that case 3 provides the smallest propagation loss in nondispersive bandwidth. The frequency dependence of the loss factor for $\beta \simeq 200$ and 90 $\mathrm{cm}^{-1}$ is shown in Fig. 6(a) and (b), respectively. In order to preserve the constant value of $\beta$, the bias field $H_0$ is altered. When $\beta$ is approximately constant, potential profiles are almost the same even though frequency varies. In the case of $\beta \simeq 200\ \mathrm{cm}^{-1}$, the potential is well confined in the first film. In the case of $\beta \simeq 90\ \mathrm{cm}^{-1}$, on the other hand, the potential peak moves toward the interface between the first and second films. It is found from Figs. 5 and 6 that propagation losses depend greatly on the line width of the film in which the potential is well confined.

## V. Conclusions

An efficient finite element solution procedure was developed for the analysis of MSW propagation losses for multilayered inhomogeneous waveguides. The final matrix equation is reduced to a standard complex eigenvalue problem whose eigenvalue corresponds to the complex phase constant itself. Thus, the phase and attenuation constants can be directly obtained by solving a standard eigenvalue equation.

The validity of the method is confirmed by calculating MSSW propagation losses for a single YIG-film structure. Also, MSBVW propagation losses for a single YIG-film structure are calculated. It is shown that the MSBVW propagation loss characteristic is similar to that for MSFVW modes. Furthermore, MSFVW propagation losses for three-layered YIG-film structure are also investigated. It is found that in the triple-layered structure, propagation losses depend greatly on the line width of the film in which the potential is well confined.

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